**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *m* = 45 minutes and *s* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: 0.2676**

**In Description:**

Let time taken for service transmission = T

T is normally distributed with mean (µ) = 45 min and Standard Deviation () = 8 min

Time delay = 10 min

Time available to finish the work = 60-10 = 50min

Therefore, from the equation Z= (T-µ)/ p= T (<=50) p= (Z<= (50-45)/8)

P (Z<= (5/8)) p(Z<=0.625) = 0.7324 (using Z table)

Therefore, p(T>50) = 1-p(<=50) = 1-0.7324 = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *m* = 38 and Standard deviation *s* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: **A) False**

68% of the data falls within one standard deviation of the mean value (+)

Here = 38 , = 6

Then = 38+6 = 44

More employees at the processing center are between 38-44.

**B) True**

Z=(X-)/ p (X<=30) p=(Z<(30-38)/6) p=(Z<(-1.33)) p= 0.0918 (using Z table)

Expected count = 0.0918 400 =36.72

At the center expected to attract about 36 employees.

1. If *X1* ~ *N (*μ, σ2) and *X*2 ~ *N (*μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if X ∼ N (μ1, σ1^2), and Y ∼ N (μ2, σ2^2) are two independent random variables then X + Y ∼ N (μ1 + μ2, σ1^2 + σ2^2), and X − Y ∼ N (μ1 − μ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above,

i.e., Z is linear combination of X and Y, then Z ∼ N (aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question 2X1~ N (2 u,4 σ^2) and

X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4 μ,6 σ^2).

The difference between 2X1 & X1+X2 is = N (4 μ,6 σ^2).

*About distributions and parameters:*

2 is simply a larger scale version of the random variable X1. If is normally distributed then 2X1 is also normally distributed.

X1 and X2 are normal distributed, the associated sums and random samples are exactly (and not just approximately) normal, with the appropriate parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:  **D. 48.5, 151.5**

**In Description:**

Here, we need range of 99% data which lies between third standard deviation

Of the mean.

Here, =100, = =20

By empirical rule, =

The values of a and b symmetric about the mean are (48.5, 151.5)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company.

Ans: μ-1.5σ = 540+((-1.5) \*(225)) =202.5 million rupees.

5th percentile of profit (in Million Rupees) is 202.5

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: percentage of (Division 1) 4.77 %

percentage of (Division 2) 4.0 %

(Division 1 has a larger probability where division 2 has

Smaller probability.)

**Division 2 has a higher probability of making loss.**